

Sourced Friedmann equations with holographic energy density

Yun Soo Myung*

*Relativity Research Center, Inje University, Gimhae 621-749, Korea
and Institute of Theoretical Science, University of Oregon, Eugene, OR 97403-5203,
USA*

Abstract

We reexamine cosmological applications of the holographic energy density in the framework of sourced Friedmann equations. This framework is suitable because it can accommodate a macroscopic interaction between holographic and ordinary matter naturally. In the case that the holographic energy density decays into dust matter, we propose a microscopic mechanism to generate an accelerating phase. Actually, the cosmic anti-friction arisen from the decay process induces acceleration. For examples, we introduce two IR cutoffs of Hubble horizon and future event horizon to test this framework. As a result, it is shown that the equations of state for the holographic energy density are determined to be the same negative constants as those for the dust matter.

*e-mail address: ysmyoung@physics.inje.ac.kr

1 Introduction

Supernova (SUN Ia) observations suggest that our universe is accelerating and the dark energy contributes $\Omega_{\text{DE}} \simeq 2/3$ to the critical density of the present universe [1]. Also cosmic microwave background observations [2] imply that the standard cosmology is given by the inflation and FRW universe [3]. Although there exist a number of dark energy candidates, the best known candidates are the cosmological constant and the quintessence scenarios. The equation of state (EOS) for the latter is determined mostly by the dynamics of scalar and tachyon.

On the other hand, there exists the dynamical cosmological constant derived by the holographic principle. Cohen *et al* showed that in quantum field theory, the UV cutoff Λ is related to the IR cutoff L_Λ due to the limit set by forming a black hole [4]. In other words, if ρ_Λ is the quantum zero-point energy density caused by the UV cutoff, the total energy of system with size L_Λ should not exceed the mass of the system-size black hole: $L_\Lambda^3 \rho_\Lambda \leq 2L_\Lambda/G$. Here the Newtonian constant G is related to the Planck mass ($G = 1/M_p^2$). If the largest L_Λ is chosen to be the one saturating this inequality, the holographic energy density is then given by $\rho_\Lambda = 3c^2 M_p^2 / 8\pi L_\Lambda^2$ with a factor $3c^2$. We consider ρ_Λ as the dynamical cosmological constant. Taking L_Λ as the size of the present universe (Hubble horizon: R_{HH}), the resulting energy is comparable to the present dark energy [5]. Even though it leads to the data, this approach seems to be incomplete. This is because it fails to recover the equation of state for a dark energy-dominated universe [6]. In order to resolve this situation, the two candidates for the IR cutoff are proposed. One is the particle horizon R_{PH} . This provides $\rho_\Lambda \sim a^{-2(1+1/c)}$ which gives the equation of state $\omega_\Lambda = 1/3$ for $c = 1$ [7]. Unfortunately, it corresponds to a decelerating universe. In order to find an accelerating universe, one needs to introduce the other known as the future event horizon R_{FH} . In the case of $L_\Lambda = R_{\text{FH}}$, one finds $\rho_\Lambda \sim a^{-2(1-1/c)}$ which may describe the dark energy with $\omega_\Lambda = -1$ for $c = 1$. The related issues appeared in Ref.[8, 9].

The above approach to dark energy have something to be clarified. Usually, it is not an easy matter to determine the equation of state for such a system with UV/IR cutoff. Actually, we have two different views of determining the equation of state for the holographic energy density. The first view is that its equation of states is not changing as the universe evolves[10, 11]. It is fixed by $p_\Lambda = -\rho_\Lambda$ initially. An important point is that the holographic energy density itself is changing as a result of decaying into other matter. According to the total energy-momentum conservation, its change must be compensated by the corresponding change in other matter sector[12]. In this case we need a source

term to mediate an interaction between two matters in the continuity equations[13]. Here we note that the EOS for the holographic energy and ordinary matter will be determined as the same negative constant by the interaction. As a result, two matters are turned out to be imperfect fluids. We call this picture as a decaying vacuum cosmology which may be related to the vacuum fluctuations[14].

The second view is that the equation of state for the holographic energy density is not fixed but it is changing as the universe expands without interaction[6, 7]. Even for being the holographic matter ρ_Λ only, its equation of state can be determined by the first Friedmann equation for $L_\Lambda = R_{\text{PH}}$ and R_{FH} . However, the equation of state with $L_\Lambda = R_{\text{HH}}$ is not determined by the first Friedmann equation[6]. It works well for the presence of holographic and ordinary matter because the energy-momentum conservation is required for each matter[7]. Recently, it is shown that this picture works even for the presence of interaction between the holographic energy and ordinary matter[15].

In this work we study the role of holographic energy density with IR cutoff in the first view of the constant EOS. The key of our system is an interaction between holographic energy and dust matter. They are changing as a result of energy transfer from holographic energy to dust matter. The sourced Friedmann equations are proposed for a macroscopic system which can describe the interaction between holographic and dust matter as the universe expands. We argue that an interaction induces an acceleration. Especially, we introduce the corresponding microscopic model to provide cosmic anti-friction which produces an accelerating phase.

In the macroscopic picture, we allow for an interaction between holographic and ordinary matter. In general, the two continuity equations are changed as[16]

$$\dot{\rho}_\Lambda + 3H(\rho_\Lambda + p_\Lambda) = q_1, \quad p_\Lambda = \omega_{\Lambda 0}\rho_\Lambda, \quad (1)$$

$$\dot{\rho}_m + 3H(\rho_m + p_m) = q_2, \quad p_m = \omega_{m 0}\rho_m. \quad (2)$$

Here $H = \dot{a}(t)/a(t)$ represents the Hubble parameter and the overdots stand for derivative with respect to the cosmic time t . $\omega_{\Lambda 0}(\omega_{m 0})$ are the initial EOS for the holographic energy (ordinary matter). The first equation corresponds to the non-conservation of the holographic matter, while the second represents the non-conservation of the ordinary matter. Even though there exist non-conservations, one requires that the total energy-momentum be always conserved. The second Friedmann equation is given by

$$\dot{H} = -\frac{4\pi}{M_p^2}(\rho_\Lambda + p_\Lambda + \rho_m + p_m). \quad (3)$$

Integrating the above equation gives the sourced Friedmann equation

$$H^2 = \frac{8\pi}{3M_p^2} \left[\rho_\Lambda + \rho_m - \int^t q_1 dt - \int^t q_2 dt \right]. \quad (4)$$

Usually the two Friedmann equations together with the continuity equation are viewed on an equal footing so that only two of them are independent. According to the thermodynamical approach to the Friedmann equations [16], the second Friedmann equation (3) is more fundamental than the first Friedmann equation (4). As a result, it is shown that Eq.(3) remains the same form even in the presence of sources but it is not always true for Eq.(4). It is important to note that both ρ_Λ and ρ_m do not evolve according to the ω_0 -parameters for their equations of state because there exists an interaction between ordinary matter and holographic matter. This causes a continuous transfer of energy from holographic energy to ordinary matter/vice versa, depending on the sign of two parameters q_1 and q_2 .

2 Microscopic mechanism for energy transfer

Let us imagine a universe made of cold dust matter with $\omega_{m0} = 0$ but obeying the holographic principle. In addition, suppose that the holographic energy density be allowed to have any equation of state. But we here allow it to have $\omega_{\Lambda 0} = -1$ for our purpose. If one chooses $q_1 = -q_2 = -\Gamma\rho_m$, their continuity equations take the forms

$$\dot{\rho}_\Lambda = -\Gamma\rho_m, \quad (5)$$

$$\dot{\rho}_m + 3H\rho_m = \Gamma\rho_m. \quad (6)$$

This means that the mutual interaction provides a decaying process. That is, this is a decay of the holographic energy component into pressureless matter with the decay rate Γ . This process is necessarily accompanied by different equations of state ω_m and ω_Λ even if they start with $\omega_{m0} = 0$ and $\omega_{\Lambda 0} = -1$. The interaction induces an accelerating expansion of the universe and determines their equations of state solely. Actually, the accelerating phase arises from a largely effective non-equilibrium pressure Π_m defined as $\Pi_m \equiv -\Gamma\rho_m/3H$ ($\Pi_\Lambda = \Gamma\rho_m/3H$). Then the two dynamic equations (5) and (6) are translated into two dissipative imperfect fluids

$$\dot{\rho}_\Lambda + 3H\Pi_\Lambda = 0, \quad (7)$$

$$\dot{\rho}_m + 3H(\rho_m + \Pi_m) = 0. \quad (8)$$

Now we introduce a microscopic mechanism to generate an accelerating universe as a result of energy transfer from the holographic energy to pressureless matter. We recall that a perfect fluid consists of particles with mass m which move on geodesics according to the geodesic equations: $m dx^i/d\tau = p^i$ and $Dp^i/d\tau = 0$, where τ denotes the proper time[17]. This corresponds to a Boltzmann equation for one-particle distribution function $f(x, p)$

$$p^i f_{,i} - \Gamma_{kl}^i p^k p^l \frac{\partial f}{\partial p^i} = C[f]. \quad (9)$$

$C[f]$ is the Boltzmann collision integral which describes elastic binary collisions between the particles. One of the second moments for distribution function is the energy-momentum tensor for a perfect fluid

$$T^{ik} = \int dP p^i p^k f(x, p) = \rho u^i u^k + p(g^{ik} + u^i u^k) \quad (10)$$

which satisfies the conservation law: $\dot{\rho} + 3H(\rho + p) = 0$. Here u^i is the macroscopic four-velocity. Actually the accelerating universe results from the cosmic anti-friction. An anti-frictional force F^i , which is arisen from the surface friction at interface between pressureless matter and holographic energy, have exerted on the particles of the cosmic substratum. As a result, the Boltzmann equation takes a different form with $Dp^i/d\tau = F^i$

$$p^i f_{,i} - \Gamma_{kl}^i p^k p^l \frac{\partial f}{\partial p^i} + m F^i \frac{\partial f}{\partial p^i} = C[f] \quad (11)$$

which shows that the individual particle motion is no longer geodesic. In a spatially homogeneous and isotropic universe, the force F^i may be given by the general difference between the macroscopic and particle velocities: $F^i = m(Bu^i - Cu_{(p)}^i)$, where B and C are not constants but should depend on the particle and fluid quantities, and $u_{(p)}^i$ is the particle four-velocity. We achieve $B = C$ only for $u^i = u_{(p)}^i$ to guarantee that the mean motion remains force-free. Under this condition, we have the microscopic form of force with the particle energy $E = -p^i u_i$

$$m F^i = B(-E p^i + m^2 u^i) \quad (12)$$

which makes the individual particles move on non-geodesic trajectories, while the macroscopic mean motion remains geodesic because for $p^i = mu^i$ and $E = m$, F^i vanishes consistently. In the case that the cosmic substratum is non-relativistic dust matter ($p_m \ll \rho_m$), the spatial projection of force is reduced to

$$e_i F^i = -B m v, \quad (13)$$

where $e^i = (p^i - Eu^i)/\sqrt{E^2 - m^2}$ is the spatial direction of the particle momentum and v is the velocity of non-relativistic particle. Eq.(13) is nothing but Stokes' law of friction. For $B > 0$, the force may be interpreted as cosmic friction but for $B < 0$, as cosmic anti-friction. The quantity B determines the strength of the force and in turn the macroscopic interaction between the holographic energy and dust matter. As was shown in Ref.[17], a microscopic force F^i -term in the Boltzmann equation leads to the source term in the balances of second moment of f as

$$\dot{\rho}_m + 3H\rho_m = -3B\rho_m. \quad (14)$$

Comparing Eq.(6) with Eq.(14) leads to $\Gamma = -3B$, which means that the (macroscopic) decay rate is given by the (microscopic) cosmic-antifricition coefficient. With the definition of the imperfect pressure $\Pi_m H = -\Gamma\rho_m/3(= B\rho_m)$, the above energy balance becomes

$$\dot{\rho}_m + 3H(\rho + \Pi_m) = 0. \quad (15)$$

This proves that the action of force manifests itself as a dissipative pressure macroscopically. In the next two section, we provides two examples which determine the quantity B by choosing an explicit form of holographic energy density.

3 Holographic energy density with the Hubble horizon

In this section we choose the IR cutoff as Hubble horizon with $L_\Lambda = R_{\text{HH}} \equiv 1/H$. Then the holographic dark energy takes the form [7]

$$\rho_\Lambda = \frac{3c^2 M_p^2 H^2}{8\pi}. \quad (16)$$

Here we consider the two interesting cases only. First we consider the non-interacting case. In this case the sourced Friedmann equation (4) with $q_1 = q_2 = 0$ can be simplified as

$$(1 - c^2)H^2 = \frac{8\pi}{3M_p^2}\rho_m. \quad (17)$$

For $c^2 \neq 1$, the first Friedmann equation takes the form of $\rho_m = 3M_p^2(1 - c^2)H^2/8\pi$. This implies that ρ_Λ behaves as ρ_m because of $\rho_m \sim H^2 \sim \rho_\Lambda$ [6, 7]. Choosing $\omega_m^{\text{HH}} = 0$ initially, one finds a dust-like equation of state for the holographic matter: $\omega_\Lambda^{\text{HH}} = 0$. This is not the case because the holographic energy density with $\omega_m^{\text{HH}} = 0$ cannot describe an accelerating universe.

Hence we study an interacting case of $q_1 = -q_2 \equiv -q$ with $\omega_{\Lambda 0}^{\text{HH}} = -1$ and $\omega_{\text{m}0}^{\text{HH}} = 0$. This case was discussed in the study of a decaying vacuum cosmology [10, 12, 18]. Here we have three equations:

$$\dot{\rho}_{\Lambda} = -q, \quad \dot{\rho}_{\text{m}} + 3H\rho_{\text{m}} = q, \quad (1 - c^2)H^2 = \frac{8\pi}{3M_p^2}\rho_{\text{m}}. \quad (18)$$

In the case of $c^2 \neq 1$, differentiating the last equation with respect to the cosmic time and then using Eq.(3) leads to $\dot{\rho}_{\text{m}} + 3H\rho_{\text{m}} = 3c^2H\rho_{\text{m}}$. Comparing it with the second equation in Eq.(18), one finds $q = \Gamma\rho_{\text{m}} = 3c^2H\rho_{\text{m}}$. Their solution are given by $\rho_{\text{m}} = \rho_{\text{m}0}a^{-3(1-c^2)}$ with the constant EOS $\omega_{\text{m}}^{\text{HH}} = -c^2$ and $\rho_{\Lambda} = \frac{c^2\rho_{\text{m}0}}{1-c^2}a^{-3(1-c^2)}$ with the EOS $\omega_{\Lambda}^{\text{HH}} = -c^2$, respectively. Here $\rho_{\text{m}0}$ is the initial value of ρ_{m} . It is noted that three equations in (18) are not enough to determine c in the holographic energy density. Introducing the dissipative pressures

$$q = \Gamma\rho_{\text{m}} \equiv -3H\Pi_{\text{m}} \equiv 3H\Pi_{\Lambda}, \quad (19)$$

one finds two imperfect fluids which satisfy

$$\dot{\rho}_{\Lambda} + 3H\Pi_{\Lambda} = 0, \quad (20)$$

$$\dot{\rho}_{\text{m}} + 3H(\rho_{\text{m}} + \Pi_{\text{m}}) = 0. \quad (21)$$

In this case the quantity B is determined as $B = -c^2H < 0$. This means that the interaction between holographic energy and dust matter induces an accelerating universe through the cosmic anti-friction if one chooses $c^2 > 1/3$. Unfortunately, the ordinary and holographic matter evolve in exactly the same way as $\omega_{\text{m}}^{\text{HH}} = \omega_{\Lambda}^{\text{HH}} = B/H = -c^2$.

4 Holographic energy density with the future event horizon

In order to find an accelerating universe which satisfies

$$\ddot{a} > 0 \leftrightarrow \frac{d}{dt}\left(\frac{1}{aH}\right) < 0 \leftrightarrow \omega < -\frac{1}{3}, \quad (22)$$

we need to take a shrinking comoving Hubble scale, as was shown in the inflationary universe. It means that the changing rate of $1/aH$ with respect to a is always negative for an accelerating universe. For this purpose, we introduce the future event horizon $L_{\Lambda} = R_{\text{FH}} \equiv a \int_t^{\infty} (dt/a) = a \int_a^{\infty} (da/Ha^2)$ [7, 8]. In this case the sourced Friedmann equation takes the form

$$H^2 = \frac{8\pi}{3M_p^2} \left[\frac{3c^2M_p^2}{8\pi R_{\text{FH}}^2} + \rho_{\text{m}} - \int^t q_1 dt - \int^t q_2 dt \right]. \quad (23)$$

Here we discuss the two interesting cases only. First we consider the non-interacting case. In order to recover the known non-interacting solution, we consider the case that $\omega_m^{\text{FH}} = 0$ for all time, while $\omega_\Lambda^{\text{FH}}$ varies as the universe evolves. In this case we find the effective EOS for the holographic energy density[7]

$$\omega_\Lambda^{\text{FH}} = -\frac{1}{3} - \frac{2\sqrt{\Omega_\Lambda}}{3c}, \quad (24)$$

where $\Omega_m + \Omega_\Lambda = 1$ with $\Omega_m = 8\pi\rho_m/3M_p^2H^2$ and $\Omega_\Lambda = 8\pi\rho_\Lambda/3M_p^2H^2$. This shows an accelerating phase. Eq.(24) is a time-dependent EOS because Ω_Λ will be determined by solving the differential equation. Further, an interacting solutions for the presence of ordinary and holographic matter appeared in[15]

Now we in a position to study the interacting case. In this case we have three equations with $\omega_{\Lambda 0}^{\text{HH}} = -1, \omega_{m 0}^{\text{HH}} = 0$:

$$\dot{\rho}_\Lambda = -q, \quad \dot{\rho}_m + 3H\rho_m = q, \quad H^2 = \frac{8\pi}{3M_p^2}[\rho_\Lambda + \rho_m]. \quad (25)$$

With $q = \Gamma\rho_m = -3B\rho_m = \epsilon H\rho_m$, the solution to the above equations is given by

$$\rho_m = \rho_{m0}a^{-3(1-\epsilon/3)}, \quad \rho_\Lambda = \frac{\epsilon\rho_{m0}}{3-\epsilon}a^{-3(1-\epsilon/3)}, \quad \epsilon = 1 + \frac{2}{3c^2} \pm \frac{2\sqrt{3c^2+1}}{3c^2}. \quad (26)$$

We observe that due to the interaction, the ordinary matter no longer scales like $\rho_m \sim a^{-3}$. In the case of $c^2 = 1$, one has $\epsilon = 1/3(-)$ and $\epsilon = 3(+)$. The first case corresponds to a decelerating universe which contains a reduced form of dust-matter with $\rho_m(\rho_\Lambda) \sim a^{-8/3}$, while the last case is an accelerating universe with the cosmological constant $\rho_m(\rho_\Lambda) \sim \text{const}$. We note that the case of $\epsilon = 1/3$ corresponds to the particle horizon even though it is derived from the future event horizon.

However, the ordinary and holographic matter evolve in exactly the same way. Hence we may confront with the same trouble as other $\Lambda(t)$ CDM cosmology[11]. Here the anti-friction coefficient B is given by $B/H = \omega_m^{\text{FH}} = -\frac{\epsilon}{3}$. Finally, the constant EOS for the future event horizon is given by

$$\omega_m^{\text{FH}} = \omega_\Lambda^{\text{FH}} = -\frac{1}{3} - \frac{2}{9c^2} - \frac{2\sqrt{3c^2+1}}{9c^2}. \quad (27)$$

5 Discussions

Introducing an interaction between the holographic energy density and dust matter, we obtain the enhanced information on the equation of state $\omega_\Lambda^{\text{HH}} = -c^2$ for the holographic

energy density $\rho_\Lambda = 3c^2 M_p^2 / 8\pi L_\Lambda^2$ with $L_\Lambda = 1/H$. Without interaction, One finds that the equation of state is fixed to be $\omega_\Lambda^{\text{HH}} = 0$ as that for a dust matter. However, the ordinary and holographic matter evolve in exactly the same way. This may induce a trouble of the indifference between the holographic energy density and ordinary matter in the future universe. In the case of the interaction between holographic energy density with the future event horizon and dust matter, we find the similiar case but with the different EOS as is shown in Eq.(27).

It is not an easy matter to determine the equation of state for a system with UV or IR cutoff. As another example, we introduce the perturbations of inflation in the early universe. In the transplanckian approach to inflation with UV cutoff Λ , the equation of state for quantum and classical fluctuations of inflation depends on the scheme of a calculation. It is usually given by $\omega_{\text{qfi/cfi}} = 1/3$ without the transplanckian backreaction. Even for quantum fluctuations of inflation, a constant energy density $\rho_{\text{qfi}} \sim (\Lambda H)^2$ with $\omega_{\text{qfi}} = 1/3, \dot{H} = 0$ is not compatible with the continuity equation: $\dot{\rho}_{\text{qfi}} + 3H\rho_{\text{qfi}}(1 + \omega_{\text{qfi}}) = 0$ [19]. To restore the compatibility, the continuity equation should be modified as $\dot{\rho}_{\text{qfi}} + 3H\rho_{\text{qfi}}(1 + \omega_{\text{qfi}}) = q$ with a source q . Then this modified equation is satisfied with $q = 4H\rho_{\text{qfi}}$. Furthermore, it gives $\omega_{\text{qfi/cfi}} \simeq -1$ when including the transplanckian backreaction with a non-linear dispersion relation [20]. On the other hand, if one includes the effects of transplanckian backreaction through the sourced Friedmann equations [16], it provides a different result of $\omega_{\text{qfi}} \simeq 1/3(1 - 4\Lambda^2/M_p^2)$.

Consequently, our system is composed of holographic energy and dust matter with their interaction $\Gamma\rho_m$. They are changing as a result of decaying from holographic energy into dust matter with decay rate Γ . The sourced Friedmann equations are suitable for a macroscopic system which can describe the interaction between holographic and dust matter as the universe expands. For clarity, we introduce the corresponding microscopic model to provide cosmic anti-friction $B = -3/\Gamma$ which plays a key role in producing an accelerating phase. Finally, we argue that an interaction induces an acceleration.

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References

- [1] S. J. Perlmutter *et al.*, *Astrophys. J.* **517**, 565(1999)[astro-ph/9812133]; A. G. Reiss *et al.*, *Astron. J.* **116**, 1009 (1998)[astro-ph/9805201]; A. G. Reiss *et al.*, *Astrophys. J.* **607**, 665(2004)[astro-ph/0402512];
- [2] H. V. Peiris *et al.*, *Astrophys. J. Suppl.* **148** (2003) 213 [astro-ph/0302225]; C. L. Bennett *et al.*, *Astrophys. J. Suppl.* **148** (2003) 1[astro-ph/0302207]; D. N. Spergel *et al.*, *Astrophys. J. Suppl.* **148** (2003) 175[astro-ph/0302209].
- [3] A. H. Guth, *Phys. Rev. D* **23**, 347 (1981); A. D. Linde, *Phys. Lett. B* **108**, 389 (1982); A. Albrecht and P. J. Steinhardt, *Phys. Rev. Lett.* **48**, 1220 (1982).
- [4] A. Cohen, D. Kaplan, and A. Nelson, *Phys. Rev. Lett.* **82**, 4971 (1999)[arXiv:hep-th/9803132].
- [5] P. Horava and D. Minic, *Phys. Rev. Lett.* **85**, 1610 (2000)[arXiv:hep-th/0001145]; S. Thomas, *Phys. Rev. Lett.* **89**, 081301 (2002).
- [6] S. D. Hsu, *Phys. Lett. B* **594**,13 (2004)[hep-th/0403052].
- [7] M. Li, *Phys. Lett. B* **603**, 1 (2004)[hep-th/0403127].
- [8] Q-C. Huang and Y. Gong, *JCAP* **0408**, 006 (2004)[astro-ph/0403590]; Y. Gong, *Phys. Rev. D* **70**, 064029 (2004)[hep-th/0404030]; B. Wang, E. Abdalla and Ru-Keng Su, hep-th/0404057; K. Enqvist and M. S. Sloth, *Phys. Rev. Lett.* **93**, 221302 (2004) [hep-th/0406019]; S. Hsu and A. Zee, hep-th/0406142; K. Ke and M. Li, hep-th/0407056; P. F. Gonzalez-Diaz, hep-th/0411070; S. Nobbenhuis, gr-qc/0411093; H. Kim, H. W. Lee, and Y. S. Myung, hep-th/0501118; gr-qc/0507010.
- [9] Y. S. Myung, *Phys. Lett. B* **610**, 18 (2005)[hep-th/0412224]; *Mod. Phys. Lett. A* **27**, 2035 (2005) [hep-th/0501023]; A. J. M. Medved, hep-th/0501100.
- [10] R. Horvat, *Phys. Rev. D* **70**, 087301 (2004) [astro-ph/0404204].
- [11] P. Wang and X. Meng, *Class. Quant. Grav.* **22**, 283(2005) [astro-ph/0408495].
- [12] D. Pavon and W. Zimdahl, gr-qc/0505020; W. Zimdahl, gr-qc/0505056;
- [13] W. Zimdahl, D. Pavon, and L. P. Cimento, *Phys. Lett. B* **521**, 133 (2001) [astro-ph/0105479].

- [14] T. Padmanabhan, astro-ph/0411044.
- [15] B. Wang, Y. Gong, and E. Abdalla, hep-th/0505069.
- [16] U. H. Danielsson, Phys. Rev. D **71**, 023516 (2005) [hep-th/0411172];
- [17] W. Zimdahl, D. J. Schwarz, A. B. Balakin, and D. Pavon, Phys. Rev. D **64**, 063501 (2001) [astro-ph/0009353];
A. B. Balakin, D. Pavon, D. J. Schwarz, and W. Zimdahl, New J. Phys. **5**, 085 (2003) [astro-ph/0302150].
- [18] E. Elizalde, S. Nojiri, S. D. Odintsov, and P. Wang, hep-th/0502082.
- [19] E. Keski-Vakkuri and M. S. Sloth, JCAP **0308**, 001 (2003) [hep-th/0306070].
- [20] R. H. Brandenberger and J. Martin, Phys. Rev. **D71**, 023504 (2005) [hep-th/0410223];